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LETTER TO THE EDITOR

On the existence of a solution for a slab critical problem

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Abstract. We prove that a discrete solution exists for the critical inverse eigenvalue problem by direct application of Gohberg's theorem.

In our earlier work (Sengupta and Ganguly 1980) we have shown that the inverse eigenvalue problem

$$\mathbb{K}_{\mathrm{ap}}(b)B(\nu)=\lambda_{\mathrm{ap}}(b)B(\nu)$$

where

$$\mathbb{K}_{\mathrm{ap}}(b)B(\nu)=\int_{\nu_{\mathrm{L}}(b)}^{1}K(\nu,\nu',b)B(\nu')\,\mathrm{d}\nu' \qquad \nu_{\mathrm{L}}(b)\leqslant\nu<1$$

has a unique solution. We note that we had to detract from the original critical problem in order to meet the positivity of the operator $\mathbb{K}$. This restriction was necessary for an elegant application of the theory of perturbation of linear operators in a Banach space—which leads in a natural way to the usual Gohberg–Atkinson theorem. Thus, the original equation was tailored to an approximate one (not very different from the original) to apply a general unified theory for a class of inverse eigenvalue problem. In this Letter, we prove that a discrete solution also actually exists for the original critical problem by direct application of Gohberg's theorem. We demonstrate this in the next section.

The equation under study is

$$B(\nu)=\int_{0}^{1}K(\nu,\nu',b)B(\nu')\,\mathrm{d}\nu' \tag{1}$$

where

$$K(\nu,\nu',b)=\tfrac{1}{2}c\bar{f}(c,\nu)X(-\nu')\exp(-2b/\nu')\nu'\left(\frac{\nu+K_0\tan[(b+z_0)/K_0]}{\nu^2+K_0^2}-\frac{1}{\nu+\nu'}\right).$$

To proceed with the analysis of equation (1) (as it is) we need the following theorem due to Gohberg. Let $\mathbb{K}(b)$ be an analytic operator-valued function in an open connected set G whose values are compact operators for $b\in\mathrm{G}$ on the Banach space X. Thus for any $\mu\neq0$, one of two possibilities must hold: (a) for every $b\in\mathrm{G}$, μ is an eigenvalue of $\mathbb{K}(b)$ or (b) except for a discrete set of values $b_K\in\mathrm{G}$, the operator $\mu\mathbb{I}-\mathbb{K}(b)$ has a bounded inverse which is defined everywhere, while $(\mu\mathbb{I}-\mathbb{K}(b))^{-1}$ has a pole at each of the points b_K.

We assume that the family of operators $\mathbb{K}(b)$ generated by the kernel $K(\nu, \nu', b)$ acts on an Hilbert space $\mathbb{L}_2(0, 1)$ of continuous positive functions. The spectrum of the operator $\mathbb{K}(b)$ will, in general, consist of continuous and residual parts in addition to the isolated point spectrum which need not be finite. However, the compactness of the operator $\mathbb{K}(b)$ eliminates the continuous and residual spectra and ensures a point spectrum only. The integral operator $\mathbb{K}(b)$ acting in $\mathbb{L}_2(0, 1)$ will be compact if the kernel has no singularities; this is true in each of the segments

$$0 \leq (b + z_0)/K_0 < \tfrac{1}{2}\pi$$

and

$$\tfrac{1}{2}n\pi < (b + z_0)/K_0 < \tfrac{1}{2}(n+2)\pi \qquad n = 1, 3, 5, \ldots.$$

In addition to this, negative values of b also cannot be allowed, as in this case the factor $\exp(-2b/\nu')$ in the kernel in the neighbourhood of $\nu' = 0$ makes it unbounded. We thus conclude that $\mathbb{K}(b)$ is an analytic operator-valued function in the open connected set G either in the range $0 \leq b < (\frac{1}{2}\pi K_0 - z_0)$ or $\frac{1}{2}K_0 n\pi - z_0 < b < \frac{1}{2}K_0(n+2)\pi - z_0$, $n = 1, 3, 5, \ldots$, whose values are compact operators for $b \in \mathrm{G}$ on $\mathbb{L}_2(0, 1)$. For our purpose, we apply Gohberg's theorem in the range $0 \leq b < (\frac{1}{2}\pi K_0 - z_0)$. This gives rise to one of two possibilities—either (i) for every $b \in \mathrm{G}$, the eigenvalue of the operator equation (1) is one or (ii) except for a discrete set of $b_K \in \mathrm{G}$, the operator $\mathbb{I} - \mathbb{K}(b)$ has a bounded inverse which is everywhere defined, while $(\mathbb{I} - \mathbb{K}(b))^{-1}$ has a pole at each of the points b_K. The operator $\mathbb{K}(b)$ is obviously a bounded operator and thus, for a non-trivial B to exist, the necessary condition is

$$\|\mathbb{K}\| > 1.$$

However, we find that, for $b = 0$, the kernel $K(\nu, \nu', b)$ is always less than one (table 1

Table 1. (*a*) $K(\nu, \nu', b)$ as a function of ν, ν' for $c = 1\cdot1$, $b = 0$.

ν \ ν'	0·0	0·2	0·4	0·6	0·8	1·0
0·0	−0·2689	−0·3697	−0·2894	−0·2387	−0·2005	−0·1674
0·2	0·0000	−0·1211	−0·1241	−0·1128	−0·0988	−0·0837
0·4	0·0000	−0·0562	−0·0634	−0·0600	−0·0531	−0·0447
0·6	0·0000	−0·0234	−0·0333	−0·0318	−0·0279	−0·0227
0·8	0·0000	−0·0133	−0·0158	−0·0149	−0·0127	−0·0097
1·0	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000

Table 1. (*b*) $K(\nu, \nu', b)$ as a function of ν, ν' for $c = 1\cdot3$, $b = 0$.

ν \ ν'	0·0	0·2	0·4	0·6	0·8	1·0
0·0	−0·3150	−0·3830	−0·2580	−0·1797	−0·1195	−0·0740
0·2	0·0000	−0·1000	−0·0782	−0·0478	−0·0185	0·0068
0·4	0·0000	−0·0326	−0·0213	−0·0034	0·0147	0·0313
0·6	0·0000	−0·0099	−0·0017	0·0100	0·0218	0·0329
0·8	0·0000	−0·0018	0·0035	0·0104	0·0172	0·0237
1·0	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000

Table 1. (c) $K(\nu, \nu', b)$ as a function of ν, ν' for $c = 1{\cdot}6$, $b = 0$.

$\nu \backslash \nu'$	0·0	0·2	0·4	0·6	0·8	1·0
0·0	−0·3773	−0·3650	−0·1702	−0·0436	0·0504	0·1233
0·2	0·0000	−0·0483	0·0147	0·0775	0·1297	0·1744
0·4	0·0000	0·0054	0·0427	0·0811	0·1135	0·1423
0·6	0·0000	0·0122	0·0349	0·0582	0·0779	0·0958
0·8	0·0000	0·0087	0·0208	0·0331	0·0437	0·0534
1·0	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000

Table 1. (d) $K(\nu, \nu', b)$ as a function of ν, ν' for $c = 2{\cdot}0$, $b = 0$.

$\nu \backslash \nu'$	0·0	0·2	0·4	0·6	0·8	1·0
0·0	−0·4700	−0·3016	0·0129	0·2179	0·3680	0·4822
0·2	0·0000	0·0402	0·1668	0·2746	0·3623	0·4321
0·4	0·0000	0·0513	0·1181	0·1769	0·2264	0·2668
0·6	0·0000	0·0317	0·0664	0·0976	0·1245	0·1468
0·8	0·0000	0·0163	0·0331	0·0485	0·0620	0·0735
1·0	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000

shows numerical values for different c and $b = 0$) and hence $\|\mathbb{K}\|$ is always less than one. It is thus concluded that one cannot be an eigenvalue of equation (1) for $b = 0$, and possibility (ii) of Gohberg's theorem must hold. This proves the existence of discrete b_K in the range $b \in (0, \frac{1}{2}\pi K_0 - z_0)$ for which the eigenvalue of equation (1) is one and thus the original critical problem has a non-trivial solution. We also have that the exact critical half-thickness b_c is less than the end-point half-thickness. Restriction of positivity is, however, necessary for an estimation of the perturbation parameter b using our method.

Reference

Sengupta A and Ganguly K 1980 *J. Phys. A: Math. Gen.* **13** 2341